A STUDY OF METHODS ABOUT ROADS NETWORK CAPACITY BASED ON VIRTUAL VERTICES MAX-FLOW

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ABSTRACT

Targeted on elevating traffic capacity of highway network, the thesis points out the deficiency in current research method of connectivity. By means of introducing max-flow algorithm of network theory and virtualizing starting and terminal points of upgraded highway network, this thesis puts forward a brand-new globally optimized algorithm of elevating traffic capacity of highway network. As a result, the proposed approaches in this thesis clarify and simplify traffic capacity of highway network with multiple starting and terminal points, and blaze new trails in theoretical significance, index concept and calculating method of connectivity, with a view to exploring and intensifying research on connectivity of highway network from a new perspective.

KEYWORDS

highway network, connectivity, network theory, max-flow

1. INTRODUCTION

The study of the traffic capacity of highway network is becoming a hot and difficult issue among many scholars. The highway network construction in China has arrived at a new crossroad: on the one hand, the foundation framework has been just constructed; on the other hand, the continuous development of economic and social requires for more optimized road network structure and higher quality and level of transport service, however, the available land resources are becoming more and more inadequate. On February 1 of this year, the Yichang Bridge at Mianchi Section of Lian-Huo Highway in Sanmenxia, Henan collapsed due to the explosion of vehicle carrying fireworks and crackers, which resulting the Lian-Huo Highway out of service in certain section, a large number of vehicles being stranded, efforts made to coordinate with surrounding provinces for bypass of vehicles. This accident fully exposed the problems existing in the traffic and load capacities of the highway network in many areas of China, which is mainly due to the low planning level. In the new era, the new problem met by the traffic administrator is how to make a better, smoother, more intensive and low-carbon highway network planning and layout and how to improve the traffic capacity of the highway network.

The connectivity is an important index and parameter to measure or evaluate the reliability of the interconnection network consisting of nodes and edges, which reflects the connection between each joint of the interconnection network and the architectural feature of the network. The larger the connectivity is, the higher the reliability of the network connectivity is. The highway network is just an interconnection network with various towns and cities as the nodes and roads at different sections as the edges. Therefore, the connectivity algorithm and indicator form an integral part of the research method on prediction of reasonable scale of the current highway network and index system on post evaluation of existing highway network.

Though the connectivity algorithm has been a relatively mature research method and is widely used by the transport authority in highway network planning or highway network post evaluation and other researches, it fails to reflect the connection conditions and actual connection strength between each node of the workable sub-network in case one node or edge of the interconnection network is in fault, which has an effect on the actual result of the traffic capacity, balance, rapidity and other indicators of the highway network and causes traffic bottleneck and jam, cutoff of arterial traffic and other significant issues. In order to avoid such issues, the author of this thesis thinks it is necessary to blaze new trails in theoretical significance, index concept and calculating method of connectivity based on the traditional theory, index concept, computing method and significance by reference to and combining the virtual vertices max-flow and minimum cut theorem of the computer network theory with a view to exploring and intensifying research on connectivity of highway network from a new perspective.

The combinatorial optimization algorithm of network max-flow and its dual – min-cut, which is with a research history of more than 40 years, is the important content of computer science and operations research and has been applied in many fields of science and engineering. For the maximum flow problem of networks, cut theorem and max-flow min-cut theorem are the theoretical basis of various algorithms of network max-flow and are with important theoretical significance. At present, the major algorithms of maximum flow problem of networks are classified into two types: 1. combinatorial algorithm. It can be further classified into augmenting path algorithm and preflow-push algorithm based on the push methods of flow in the residual network, among which the augmenting path algorithm includes Ford-Fulkerson's labeling algorithm, Dinic's blocking flow algorithm and Ahuja-Orlin's shortest augmenting path, etc. and the preflow-push algorithm includes Karzanov's blocking flow algorithm, Goldberg-Tarjan's push-relabel algorithm, Goldberg-Rao's blocking flow algorithm, etc. 2. Linear programming algorithm. In fact, the maximum flow problem is a special linear programming problem. In consideration of such particularity, a method such as network simplex algorithm and network interior algorithm may be obtain, which is more effective than the common linear programming algorithm.

Applying the network max-flow theory to the highway network planning, we can find the cut set with minimum capacity (the network max-flow) from all highway network cut sets based on the definition of cut set and max-flow min-cut theorem and with the virtual vertices as the data organization form.

2. OVERVIEW OF CURRENT RESEARCH METHOD OF CONNECTIVITY

2.1 Connectivity Algorithm for Prediction of Highway Network Scale

The connectivity algorithm, also known as "node model algorithm", is a method to plan the connectivity between each node within certain region via the highway based on the network geometry analysis. This method reflects the structure features of the highway network from its layout by investigating the connectivity of each traffic network node. The formula for calculating the reasonable scale of highway network with connectivity is as follows:

$$L=C\bullet\xi\bullet\sqrt{N\bullet A}$$

Formula (1)

Where:

L represents the scale of highway network (i.e. total distance of highway within the planning area);

C represents the connectivity of highway network;

ξ represents the non-linear coefficient (also known as deformation coefficient, it is the ratio of total distance of actual path between each node to the total linear distance; the value is related to the winding degree of the highway and the geometric shape of node distribution)

A represents the area of planning region;

N represents the number of nodes.

If the administrative division and area of region are relatively stable, the significant factors having influence on highway network scale are C-connectivity andξ-non-linear coefficient. Among them,ξis mainly influenced by the terrain of the region and the level of highway technology. If the region is with more hills and the highway technology level is lower, the value ofξis larger; vice versa. Generally, measurement shall be conducted to determine the deformation coefficient of the original highway network in such region for reference.

2.2 Connectivity Index for Evaluation on Connectivity of Highway Network

We can obtain the formula of connectivity-C with Formula (1) in the preceding paragraph:

$$C=\frac{L}{\xi\sqrt{N\bullet A}}$$

Formula (2)

For the physical significance, if C=1.0, it means the layout of highway network is in tree structure and two-way connection is adopted between each node; if C=2.0, it means the layout of highway network is grid-like and four-way connection is adopted between each node; if C=3.14, it means the layout of highway network is in grid-like and diagonal structure; if C=3.22, it means the layout of highway network is in regular triangular structure and six-way connection is adopted between each node. If C≤1.0, it means the highway network is with poor connectivity; if 1.0<C≤2.0, it means the highway network is with good connectivity; if 2.0<C≤3.14, it means the highway network is relatively perfect for flat area and is mature for a mountainous area; if C>3.14, it means the highway network is mature for flat area; and if C>3.22, it means the highway network is with an ideal connectivity.

Besides, the indexes for evaluation of highway network connectivity also include “α index”, “β index”, “γ index”, etc.:

$$\alpha=\frac{E-N+1}{2N-5}$$

Formula (3)

$$\beta=\frac{2E}{N}$$

Formula (4)

$$\gamma=\frac{E}{3N-6}$$

Formula (5)

Where:

E represents the edges of highway network (i.e. the number of highway linking each node in the highway network).

They reflect the highway network connectivity from different perspectives.

2.3 Deficiencies of Existing Methods

Though the connectivity method is a relatively mature research method, it is still with some deficiencies. As is shown in Fig.1, when we measure the connectivity of highway network model N_1 and N_2 with connectivity method, we may find that C_1=C_2=7/6 which indicates that the connectivity of such two highway networks are same. However, if we remove one of the edges in N_1, we can see that the Network N_1 remains connected; if we remove one of the edges in N_2, we can see that the Network N_2 will be disconnected. In fact, we can directly see that N_1 and N_2 have obvious difference in structure, especially a significant connectivity. However, the connectivity index C fails to reflect such obvious difference.

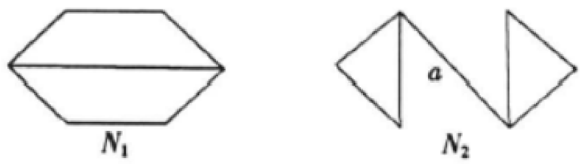

FIGURE 1 Highway Network

What is more, when we measure the highway network models N_1 and N_2 with other connectivity indexes such as "αindex", "β index", "γ index", etc., we still find that the connectivity of the two highway networks is same (see Table 1), which fails to distinguish the two networks and reflect the network structures. As a result, we should explore and intensify the research on connectivity of highway network from a new respective to blaze new trails in theoretical significance, index concept and calculating method of connectivity and improve the planning quality and level of highway network and evaluate the connectivity of highway network more scenically and objectively.

TABLE 1 Evaluation Index of Highway Network N_1 and N_2

Highway Network	αindex	βindex	γ index	Connectivity C
N_1	2/7	7/3	7/12	7/6
N_2	2/7	7/3	7/12	7/6

3. DESCRIPTION OF NETWORK MAX-FLOW ALGORITHM BASED ON VIRTUAL VERTICES

As noted in the introduction, the problem of max-flow is just a special problem of linear programming. In network, as the maximum flow (i.e. capacity) of each edge is limited and the actual flow may not equal to the capacity, the max-flow algorithm aims to take full advantages of the network capacity and realize global optimization so as to achieve the best effect (i.e. to achieve maximum flow), which is called problem of max-flow. It also aims to solve the problem how to realize global optimization and achieve maximum flow by introducing the network max-flow theory and algorithm.

3.1 Mathematical Description to Highway Network with Multiple Starting and Terminal Points

It is well known that the actual highway network doesn't have a unique or designated starting or terminal point, and each staring point or terminal point is randomly selected, which indicates such complex network characteristics of the highway network as non-linearity, stochastics & openness and multiple starting and terminal points. We may abstract the highway network with multiple staring and terminal points into graph, analyze the traffic characteristics of the highway network graph with the knowledge of graph theory and regard it as a weighted directed graph. In the graph, V_i (i=1,2,…n) represents the intersections of the highway network, V_{sj} (j=1,2,…n) represents the *m* starting point of the highway network, and V_{tk} (k=1,2,…n) represents the *q* terminal point of the highway network; the directed edges of the graph represent the sections of the highway network and are marked as e_{ij}, *i* represents the starting point of the section, *j* represents the terminal point of the section; mark the set of all sections as E and the highway network as *G(V,E)*. For the highway network *G(V,E)*, *C(e)=C(,)* - the maximum allowed traffic flow of each e_{ij} is marked as c_{ij}.

3.2 Capacity of Connectivity of the Highway Network with Multiple Starting and Terminal Points

For highway network *G(V,E)*, we add two virtual vertices v_s and v_t, connect v_s to all starting points in the highway network via forward sections and connect all terminal points to v_t via forward sections. And then regard v_s as the single starting point of new highway network *G'* and v_t as the single terminal point of the new highway network *G'*, and designate the maximum allowed traffic flow of the added virtual section as positive infinity *(M)*. In such way, we change the highway network with multiple starting and terminal points into the highway network *G'* with a single starting point and terminal point, and convert the problem into the network max-flow algorithm with single starting and terminal point.

3.2.1 Traffic Flow of Sections in Highway Network

For the highway network *G(V,E)*, we define *f(e)=f(,)* and suppose that *f(e)* is the traffic flow of section $e=(v_i, v_j)$, and mark it as f_{ij}. If *v(f)* is used to express the traffic flow from v_s to v_t *and satisfies:*

$$0 \leqslant f_{ij} \leqslant c_{ij}$$ Formula (6)

$$\sum_i f_{ij} - \sum_j f_{ij} = \begin{cases} v(f), i = s \\ 0, i \neq s, t \\ -v(f), i = t \end{cases}$$ Formula (7)

v(f) is regarded as the feasible flow of highway network *G(V,E)* from v_s to v_t; formula (1) is the maximum allowed traffic flow of any section and formula (2) is the balance condition of traffic flow.

3.2.2 Adjustment Value of Path Traffic Flow

To determine one directed path from v_s to v_t, i.e. the direction of all sections is same as that of such path. Based on the maximum allowed traffic flow, c_{ij}, of each section along such path and the section traffic flow of the highway network, the adjustment value Δf of such path traffic flow may be determined in accordance with the following formula:

$$\Delta f = \min_{e_{ij} \in p_{ij}} (c_{ij} - f_{ij})$$ Formula (8)

In Formula (3), p_{ij} is the set of all sections along this path and the initial value of f_{ij} is 0. For the path from v_s to v_t in the highway network *G(V,E)*, the maximum value of Δf is marked asΔ ∗, this path is also called as maximum feasible adjusted path.

3.2.3 Simplification of highway network

We plus the traffic flow of all sections along the path withΔf to find the one with maximum traffic flow and delete such section so as to simplify the highway network and modify the weight of section in the highway network.

$$f_{ij} = \begin{cases} f_{ij} + \Delta f, & f_{ij} \in p_{st} \\ f_i & \text{other} \end{cases}$$ Formula (10)

3.3 Strategy of Algorithm

We find all directed paths from v_s to v_t, calculate the adjustment value of each path and simplify the highway network from the path with a maximum adjustment value, and repeat the calculation in the simplified highway network until there is no directed path from v_s to v_t in the highway network *G(V,E)*. At this time, the traffic assignment of each section is the maximum traffic flow from the starting point v_s to the terminal point v_t and is just the maximum traffic flow of the whole highway network, which can effectively ensure the capacity of connectivity and maximum loading capacity of the highway network.

3.4 Steps of Algorithm

(1) To change the highway network to one with single starting and terminal point, and set the current traffic flow of each section to 0;

(2) To identify the path with maximum adjustment value from v_s to v_t;

(3) To simplify the highway network;

(4) To repeat step two until there is no directed path in the highway network;

(5) To accumulate the traffic flow of all sections with v_s as starting point and calculate the maximum traffic flow of the highway network.

4. CONCLUSION

The connectivity of highway network is the important basis of transport planning and research department for decision-making and post-evaluation, provides significant management and decision basis for provision of highway network planning quality, preparation of traffic management policies, control of traffic construction scale, etc. and has influence on the comprehensive economic benefit of the highway network and social benefit of the region. To maximize the utilization of highway resources, minimize land occupation and improve the connectivity of highway network is the direction of Chinese communication and transportation construction and development. The improvement of connectivity of highway network can better satisfy the following two basic requirements for highway traffic:

(1) Adaption and comfort. It is to improve the service level of highway network effectively to change the “single connection” of highway network to “multi-connection”.

(2) Balance and rapidity. It is to intensify the carrying capacity and increase the traffic flow of the highway network in unit time.

Compared to the traditional research method, the network max-flow algorithm based on virtual vertices is more complex; however, as popularization of computer, such innovative approach is more suitable for the solution of maximum capacity of traffic network with computer, involves less uncertain factors and is an effective algorithm.

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